$$g_n = A_n \sinh\left(\frac{n\pi}{2a} h_3\right)$$

and

$$F_n = 4 \sin\left(\frac{n\pi}{2a}\frac{w_i}{2}\right) \sin\left(\frac{n\pi}{2a}\frac{w_j}{2}\right) \begin{matrix}\cos\\ \sin\end{matrix}\left(\frac{n\pi}{2a} x_i\right) \begin{matrix}\cos\\ \sin\end{matrix}\left(\frac{n\pi}{2a} x_j\right)$$

where x_i is the x coordinate of the ith substrip.

In the above equations, the upper notation applies to a magnetic wall boundary condition at $x=0$; the lower notation applies to an electric wall. As g_n converges fairly rapidly on g_∞, (A-1) can be rewritten as

$$P_{ij} = \frac{1}{w_i w_j}\left(\frac{2a}{\pi}\right)^2 \left\{ \sum_{\substack{n=1,3\\ \text{or}\\ n=2,4}}^{N} \frac{(g_n - g_\infty)}{n^3} F_n + g_\infty \sum_{\substack{n=1,3\\ \text{or}\\ n=2,4}}^{\infty} \frac{F_n}{n^3} \right\} \tag{A-2}$$

where N is taken to be large enough for g_n to be regarded as g_∞. Since the second term in (A-2) can be analytically obtained and $(g_n - g_\infty)$ reduces to zero rapidly as n increases, (A-2) shows much faster convergence than (A-1).

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Interdigitated Microstrip Coupler Design

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Abstract—**A design procedure for four-line interdigitated couplers is presented which provides excellent agreement between performance and actual coupler dimensions. The inclusion of a correction term for the finite metal thickness of the microstriplines is significant. Using existing odd-and-even mode impedance data of only two coupled lines in the array actual coupling coefficients in the 2.5–6.5-dB range are predictable to within ±0.05 dB. Graphs are shown which relate fabrication tolerances of dielectric constant and physical line dimensions to deviations in coupling and characteristic coupler impedance. The design was verified on 3-, 5-, and 6-dB couplers in the 1–5-GHz frequency range.**

I. Introduction

THE INTERDIGITATED 3-dB coupler as described by Lange [1] is a quadrature coupler and is well-suited for realization in microstrip form. The main advantages are its small size and the relatively large line separation when compared with the gaps of a conventional two-coupled line device and its relatively large bandwidth when compared with branch-line couplers. In the 3-dB form, it is an ideal component for balanced MIC amplifiers and mixers, and for binary power divider trees.

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Interdigitated couplers can be fabricated with coupling coefficients other than 3 dB and, therefore, can be used as components for serial type dividers with power divisions other than binary. There is a need for reliable design procedures that lead to producible couplers and predictable performance.

The original description of the interdigitated coupler [1] did not provide any design information in terms of the known coupled line parameters of a line pair or the parameters derived from a rigorous charge distribution of the four-line set. A more recent publication [2] presents design equations for such couplers with an arbitrary even number of coupled lines. The equations are written in terms of even- and odd-mode impedances of only two adjacent coupled lines in the array which are identical to any other pair in the structure. These equations together with published [3] microstrip data were used in a coupler design. The basic design assumes zero conductor thickness. The physical dimensions as determined by the basic design resulted, however, in overcoupled responses of fabricated couplers in the 3- to 6-dB range on two types of substrates, namely, alumina and BeO. A metallization thickness correction applied to the line and gap dimensions, similar to those described in [4], gave better results.

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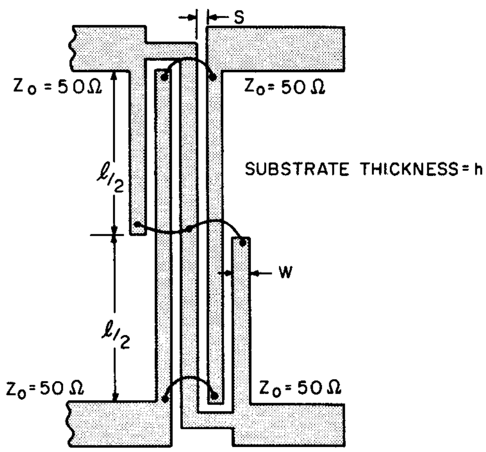

Fig. 1. Interdigitated coupler.

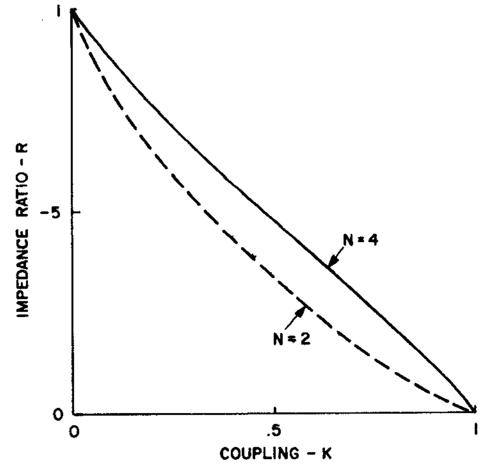

Fig. 2. Impedance ratio versus coupling.

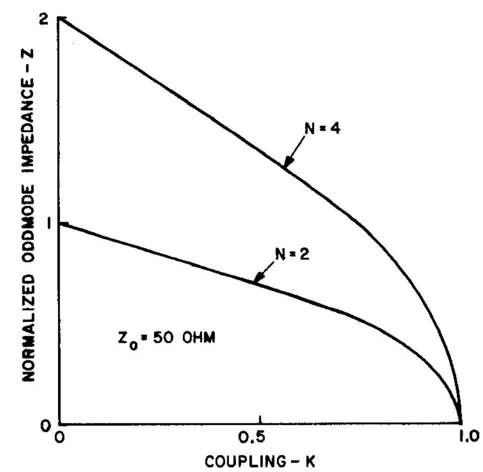

Fig. 3. Normalized odd-mode impedance versus coupling.

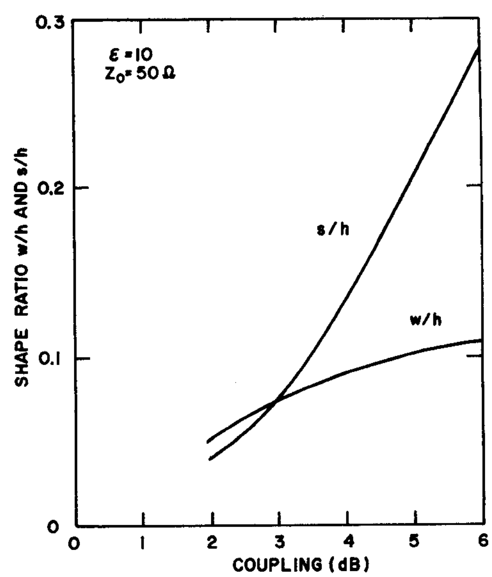

Fig. 4. Nominal shape ratios versus coupling.

The design accuracy was demonstrated with the check of at least 100 couplers with coupling coefficients ranging from 2.5 to 6.5 dB. These coefficients were predictable to within ± 0.05 dB when actual line and gap dimensions were considered.

II. Coupler Design

The physical layout of a four-line coupler is shown in Fig. 1. The design of the coupler proceeds from the specification of the desired coupling coefficient (k) and the characteristic impedance (Z_0). The coupling region (l) is a quarter wavelength at the center frequency of the operating band. The objective now is to find the gap spacing (s) and the line width (w) for a given substrate of thickness (h) and with dielectric constant (ϵ_r).

The two basic equations for an N-line coupler (N even) can be expressed as

$$K=\frac{(N-1)(1-R^2)}{(N-1)(1+R^2)+2R} \quad (1)$$

$$Z=\frac{Z_{0o}}{Z_0}\,\frac{\sqrt{R[(N-1)+R][(N-1)R+1]}}{(1+R)} \quad (2)$$

$$R=\frac{Z_{0o}}{Z_{0e}}$$

in which Z_{0o} is the odd-mode impedance and Z_{0e} is the even-mode impedance of two-coupled lines. The impedance ratio R is plotted as a function of the coupling coefficient in Fig. 2 and the normalized ($Z_0=50\ \Omega$) odd-mode impedance Z is plotted in Fig. 3. Knowing the impedances Z_{0o} and Z_{0e}, the shape ratios w/h and s/h can be obtained from known coupled-line data [3] for a given dielectric. There exists one unique set of shape ratios for which both the coupling and the system impedance relationships are met simultaneously. The necessary shape ratios for 50-Ω couplers on $\epsilon_r=10$ material were collected as a function of coupling and are shown in Fig. 4. Also computed and shown in Fig. 5 are the shape ratios for a 50-Ω 3-dB coupler as a function of substrate dielectric constant over the ϵ_r range from 2 to 16.

Applications of this zero thickness conductor design procedure described so far consistently resulted in couplers with overcoupled characteristics. It was then experimentally determined that conductor metallization thick-

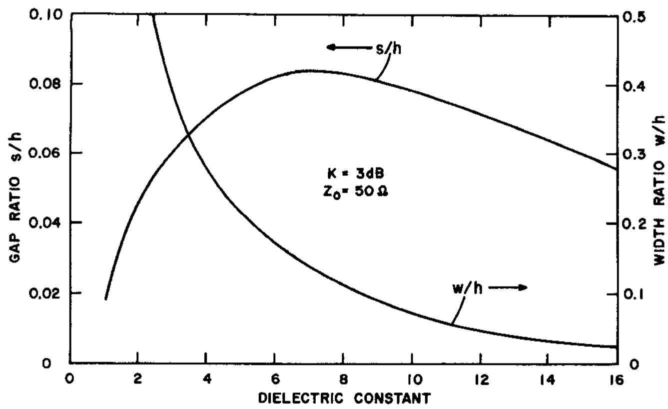

Fig. 5. Nominal 3-dB shape ratios versus dielectric constant.

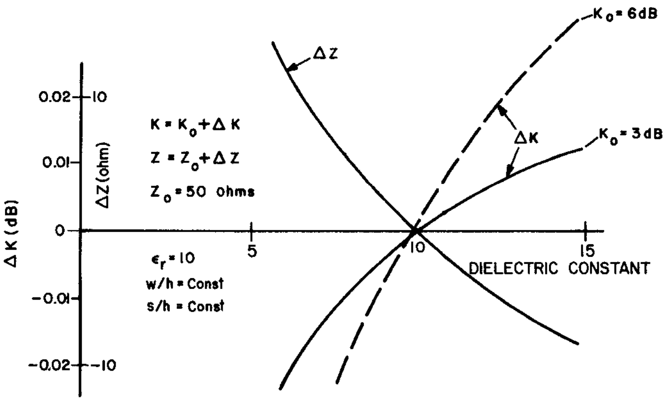

Fig. 7. Coupling and impedance deviation versus dielectric constant (shape ratio constant).

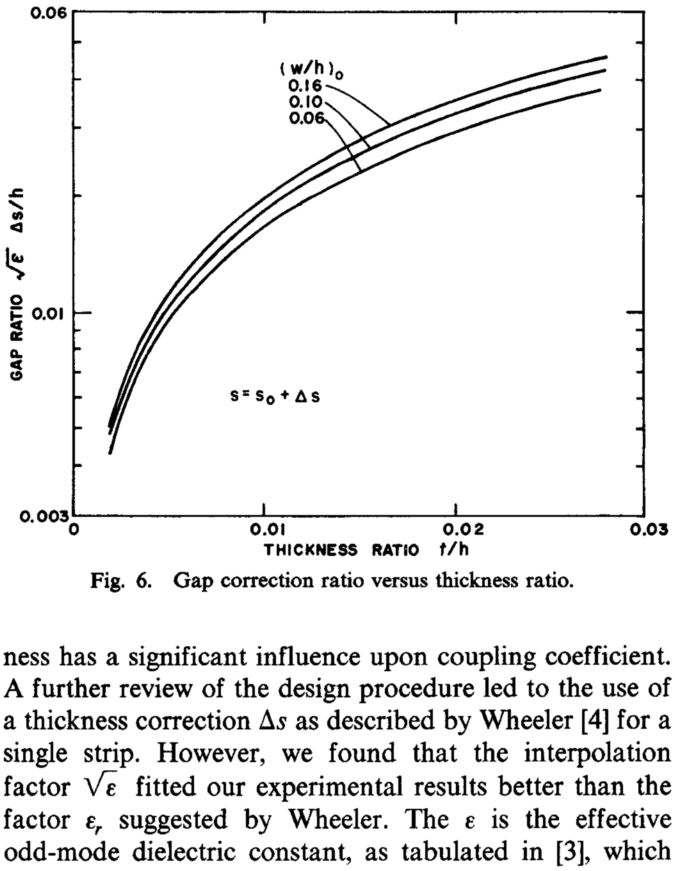

Fig. 6. Gap correction ratio versus thickness ratio.

ness has a significant influence upon coupling coefficient. A further review of the design procedure led to the use of a thickness correction Δs as described by Wheeler [4] for a single strip. However, we found that the interpolation factor $\sqrt{\varepsilon}$ fitted our experimental results better than the factor ε_r suggested by Wheeler. The ε is the effective odd-mode dielectric constant, as tabulated in [3], which corresponds to the zero thickness shape ratios. Thus the correction factor Δs was expressed in the form of

$$\Delta s/h = \frac{t/h}{\pi\sqrt{\varepsilon}}\left(1+\log_e \frac{4\pi w_0/h}{t/h}\right) \qquad (3)$$

where t/h is the actual thickness ratio of the metallization. The major effect of t is that it reduces effectively the zero thickness design gap s_0 by Δs and increases the zero thickness design width w_0 by the same amount. The plot in Fig. 6 shows the correction ratio as a function of practical thickness ratios for a wide range of zero thickness width ratios. If a tabulated value of ε is not available, it can be substituted by the effective dielectric constant ε_{eff} of a single strip with a shape ratio w_0/h; a correction factor error of at most 5 percent results for all practical coupling coefficients and substrate materials.

The described procedure was used in the design of 3-, 5-, and 6-dB couplers on alumina and on BeO substrates with a thickness of both 0.635 and 1.27 mm. Photomasks and fabrication processes were coordinated to result in actual line and gap dimensions for a nominal metallization thickness of 4.5 μm. The described coupling deviation from design values on about 150 couplers constructed was never larger than ±0.25 dB. The larger deviations were always traceable to unwanted dimensional variations that were introduced during processing. A cross check between actual dimensions and the design procedure showed agreement to within ±0.05 dB when measured dissipative losses were accounted for. Couplers at center frequencies between 1.3 and 4.5 GHz had typically over 40-percent bandwidth isolations of 25 dB, return losses greater than 20 dB, and insertion losses of 0.2 dB ± .05 dB. Attempts to compare these results with four-strip impedance solutions [5] failed since no thickness correction data for the latter approach were available.

III. Fabrication Tolerances

The effects of fabrication tolerances are expressible in terms of coupling (k) and characteristic impedance (Z_0) sensitivities. Generally, sensitivity functions [6] are easily derivable from (1) and (2). However, they would relate to the odd- and even-mode impedance changes only and not directly to the manufacturable dimensions. Therefore, coupling and impedance changes were calculated, using the available odd- and even-mode impedance data [3], as a function of dielectric constant and of fractional changes in gap and linewidth. One parameter was changed at a time, while all other parameters stayed fixed at their nominal values.

The graph in Fig. 7 shows both coupling and impedance deviations as a function of dielectric constant over the range of $\varepsilon_r = 5$ to $\varepsilon_r = 15$. The initial designs are assumed to be for 50-Ω couplers on alumina for 3- and 6-dB coupling. The impedance variations are essentially independent of coupling and near the desired value of $\varepsilon_r = 10$; a 10-percent change in dielectric constant corresponds to a 5-percent change in characteristic impedance. Coupling deviations are extremely small since for a given

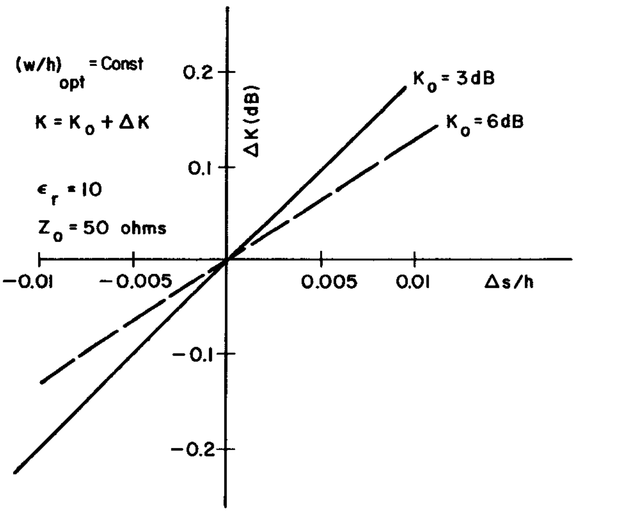

Fig. 8. Coupling deviation versus gap deviation (width ratio constant).

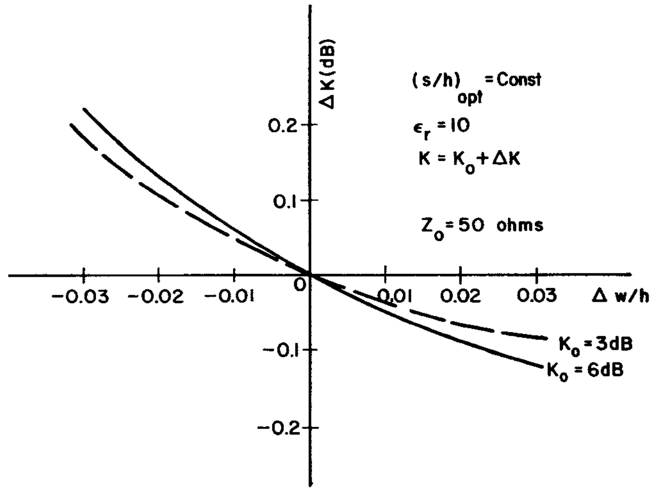

Fig. 10. Coupling deviation versus gap deviation (gap ratio constant).

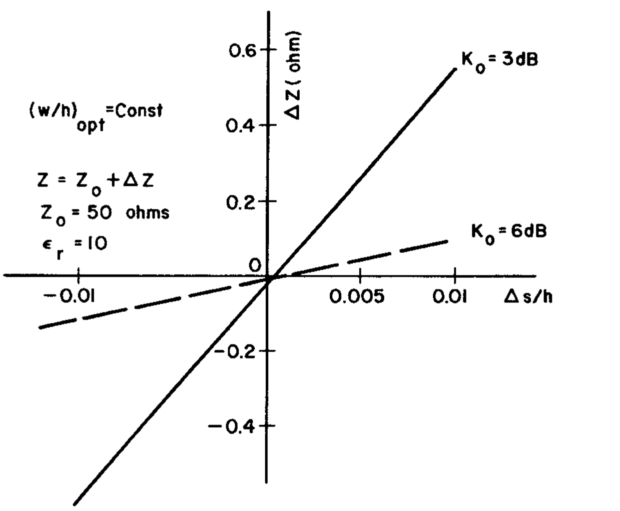

Fig. 9. Impedance deviation versus gap deviation (width ratio constant).

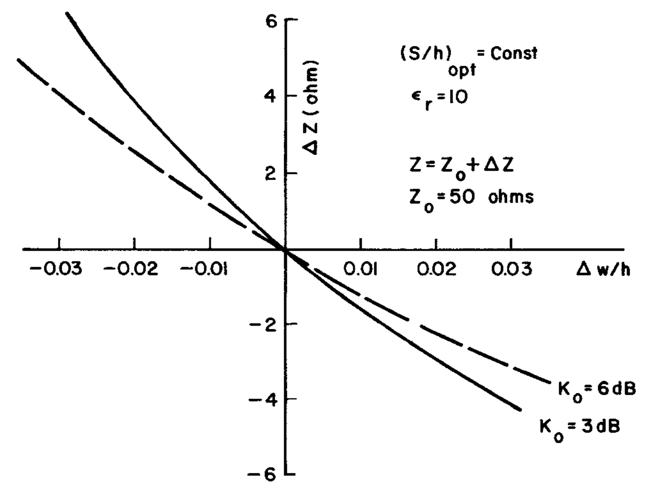

Fig. 11. Impedance deviation versus gap deviation (gap ratio constant).

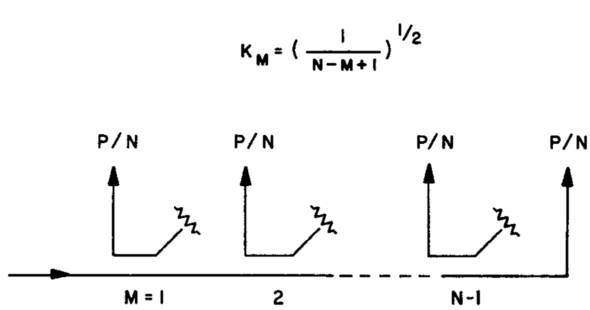

Fig. 12. Serial power divider schematic.

shape ratio pair (w/h and s/h) the impedance ratio R is essentially independent of ε_r over the range of interest, and coupling is dependent upon R only (see (1)).

Figs. 8 and 9 depict the coupling deviation and impedance variation, respectively, as a function of the normalized gap change from nominal while the width remains unchanged. The dominant effect here is the coupling deviation. Assuming a 0.635-mm thick substrate the coupling sensitivity is about 0.03-dB/μm gap variation for a 3-dB coupler and 0.02 dB/μm for a 6-dB coupler. Impedance deviations from 50 Ω are extremely small amounting to 80 mΩ/μm for a 3-dB coupler.

For the case in which the gap remains constant and the line width is allowed to vary, coupling deviations are negligible (0.01 dB/μm), whereas impedance deviations are high (250 mΩ/μm). Figs. 10 and 11 show the corresponding plots for this case. It is worthwhile to note that for a 3-dB coupler a 40-percent increase in line width only changes the coupling by about 0.1 dB whereas the impedance decreases to 45 Ω causing a 1.1 VSWR.

IV. Design Example

The above described design procedure was used on the design of a four-way serial divider. A serial divider [7] for N equal power divisions consists of a serial cascade of $(N-1)$ couplers each having a different coupling coefficient. This scheme, as depicted in Fig. 12, provides N integer power divisions with all desired performance features of quadrature couplers when four-line interdigitated couplers are used as building blocks. The four-way divider requires three couplers with power coupling coef-

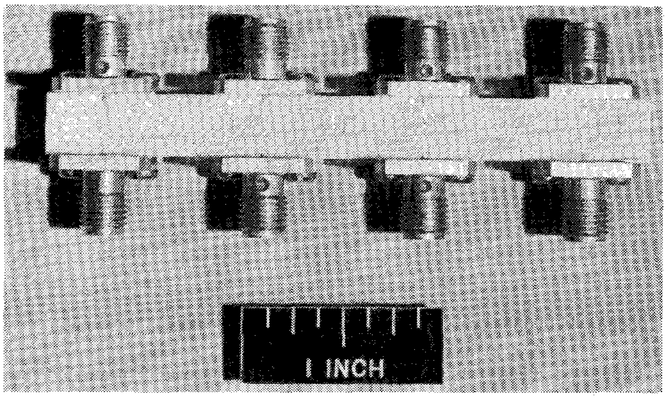

Fig. 13. Photograph of four-way serial divider.

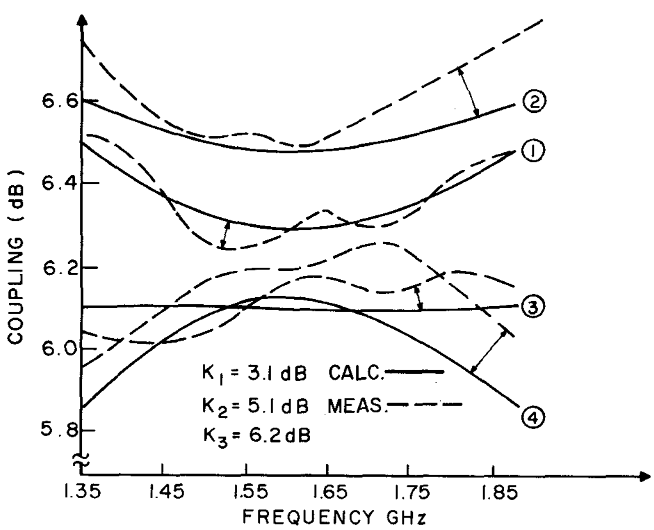

Fig. 14. Performance comparison of four-way serial divider.

ficients (K^2) of 1/4, 1/3, and 1/2, respectively. The coupler was fabricated on a single 1.27-mm thick alumina substrate with all couplers directly interconnected. Fig. 13 shows a photograph of such a coupler for the 1.35–1.85-GHz frequency range. From the final dimensions the coupling coefficients of the component couplers were calculated and found to be 3.1, 5.1, and 6.2 dB, respectively. Using these values and assuming a dissipative loss of 0.1 dB per coupler, the responses of the four outputs were calculated over the 1.6-GHz ± 15-percent frequency band. These results, together with the actually measured values, are shown in Fig. 14. Especially near the band center, agreement is very good. The measured return loss of the coupler was in excess of 28 dB, and the isolation was greater than 30 dB. These results substantiate that coupler responses can be accurately predicted, and that deviations from design goals are mainly caused by dimensional variations that arise during fabrication.

V. Conclusion

The described procedure, even though based upon an approximate description of a four-line interdigitated coupler [2], together with published coupled-line data [3] and an empirical thickness correction term, can be used to design producible couplers in the 2.5–6.5-dB coupling range. Tolerance studies indicate that the most sensitive parameters are the gap dimensions and the metallization thickness, whereas width and dielectric constant changes of 10 percent cause practically insignificant deviations.

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